MULTICHANNEL SAMPLING OF SIGNALS WITH FINITE RATE OF INNOVATION USING EXPONENTIAL SPLINES

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1. ABSTRACT

Consider a multichannel sampling system consisting of many acquisition devices observing an input finite rate of innovation (FRI) signal, a non-bandlimited signal that has finite number of parameters [1, 2]. Each acquisition device has access to a delayed version of the input signal where the delays are unknown. By synchronizing the different channels exactly we are able to reduce the number of samples needed from each channel resulting in a more efficient sampling system. Figure 1 shows the described multichannel sampling system where the bank of acquisition devices $\varphi_1(x,y), \varphi_2(x,y), \ldots, \varphi_{N-1}(x,y)$ receive different versions of the input FRI signal $g_0(x,y)$. Here, the delays (1-D) or geometric transformations (2-D) are denoted by $\mathcal{T}_1, \mathcal{T}_2, \ldots, \mathcal{T}_{N-1}$.

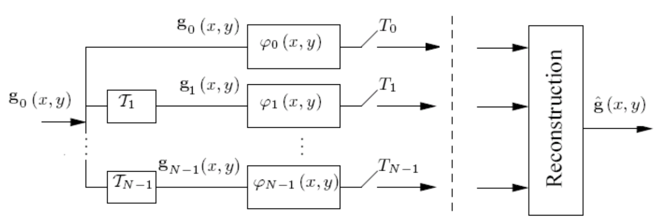

Fig. 1. Multichannel sampling setup

In [4] Baboulaz considered the use of exponential splines (E-splines) [3] for sampling a stream of 1-D Dirac impulses in a multichannel sampling setup described in Figure 1. An advantage of E-spline sampling kernels is that they can be employed in a fully symmetric multichannel sampling environment. By symmetric sampling, we mean that the sampling process can be evenly distributed between different acquisition devices. It was shown that if two 1-D signals are just shifted version of the other, then by setting one parameter to be common between the exponents of the E-spline sampling kernels for the two signals, one can not only estimate the shifts between the two signals, but also can linearly relate the exponential moments of the two signals (the reader can refer to [4, 5] for a more detailed discussion). Because of the direct relationship between the exponential moments of the two signals, we can achieve perfect reconstruction of the reference signal with fewer exponential moments required. Since less moments are required from each channel, a lower order E-spline sampling kernel would be needed, which in turn less samples from each signal are required to achieve perfect reconstruction. This is because, from [2] we know that a stream of Dirac impulses is uniquely determined from the samples if there are at most K Dirac impulses in an interval size of $2KLT$ where L is the support of the sampling kernel. Since the support of the sampling kernels is reduced in the multichannel case, we can achieve the same performance with a smaller sampling rate T. For the 2-D case, in [5] we illustrate that symmetric multichannel sampling of bilevel polygons (a 2-D FRI signal) can be achieved with the geometric transformation being a 2-D translation between the different signals. For the case of more complicated transformations such as scaling and rotation, we can not estimate the parameters like the way it was done for the simple translation case in [5] with exponential reproducing kernels. Also, even if we assume that the transformation parameters are known and given, we still can not use the sampling algorithm shown in [5] for the multichannel framework. This is because introducing more complicated transforms such as rotation and/or scaling for example, would result in a non-linear relationship between the exponential moments of the different signals.

The first question we need to answer is that, assuming an oracle gives us the values of the transformation parameters, can we sample and perfectly reconstruct translated, rotated and scaled bilevel polygons in a symmetric multichannel framework? It is known that for an N-sided bilevel polygon, with N+1 projections, perfect reconstruction of the polygon can be achieved. That is points that have N+1 line intersections from the N+1 back-projections correspond to the N vertices of the polygon. We also know that a Radon projection at an angle ϕ of a rotated image with respect to its reference image with an angle θ, is the same projection, but scaled and translated, on the reference image at the angle $\phi + \theta$. Therefore, if all the transformation parameters are known, and assuming that the rotation angle is not zero that is, $\theta \neq 0$, then the $N + 1$ projections needed could be separated between the different channels, in order to sample and perfectly reconstruct the reference image in a symmetric manner. The next question would be, how can we estimate the transformation parameters? We know that with the use of polynomial reproducing kernels, we can obtain the geometric moments of a signal, and geometric moments up to order 2 from two signals are enough to estimate translation, rotation and scaling parameters between the two signals. We also know that, as E-splines are a generalized version of B-splines [3], we can reproduce a combination of polynomials and exponentials from E-splines. From the polynomials moments up to order 2, we can estimate all the transformation parameters.

2. RESULTS

As an example, in [5] we showed that to achieve perfect reconstruction for a 4-sided bilevel polygon, a 2-D E-spline order of 12 is required to produce 5 projections at the angles $0, 45, 90, tan^{-1}(2)$ and $tan^{-1}(\frac{1}{2})$. With 2-D E-spline order of 7 however we can produce 3 projections at the angles $0, 45, 90$ on the reference signal, and a 2-D E-spline order of 7 on the second signal would give 3 projections for the reference signal at the angles $\theta, 45 + \theta, 90 + \theta$ where θ is the rotation parameter. Assuming θ is not zero, we would have enough projections to perfectly reconstruct the reference signal. Therefore an spline order of 7+2 = 9 (2 is needed for estimating the transformation parameters) on each signal would give us enough projections to perfectly reconstruct the reference signal. An example for a 4-sided bilevel polygon with two acquisition devices is shown in Figure 2 where the reference signal, its translated, rotated and scaled version, their samples, the E-spline sampling kernel, and the reconstructed reference signal are all shown.

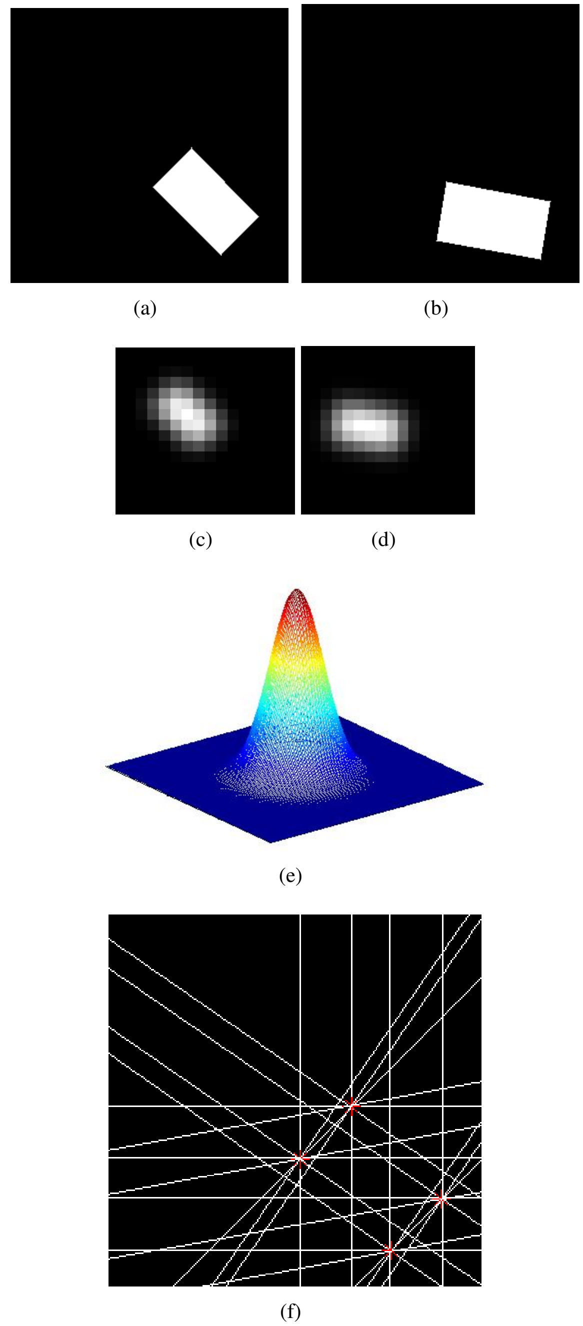

Fig. 2. Symmetric multichannel sampling of translated, rotated and scaled bilevel polygons using E-spline sampling kernels. (a) The reference signal in a frame data size of 256×256. (b) The translated $(\triangle x = -100, \triangle y = 150)$, rotated $(\theta = 35)$ and scaled $(a = 1.1)$ version of the reference signal. (c) & (d) The 16×16 samples of both signals. (e) 2-D generalized E-spline of order 9 (f) The reconstructed vertices of the reference signal with 6 back-projections, the crosses are the actual vertices of the polygon. [Not to scale]

3. REFERENCES

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